SIMILAR SOLUTION OF THE PROBLEM OF GAS MOTION IN A POROUS MEDIUM TAKING HEAT EXCHANGE INTO ACCOUNT

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The article presents the similar solution of the problem of gas temperature and pressure when the gas moves in a porous medium with linear law of resistance.

Similar solutions of problems concerned with gas motion in a porous medium are dealt with in a number of works (see, e.g., [1-5]). Below we examine the similar solution with a view to the heat exchange of a gas with the surface of a porous medium.

The system of equations describing such motion has the form [6]

$$m_0 \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

$$m_0 \frac{\partial (\rho e)}{\partial t} + m\rho_s \frac{\partial e_s}{\partial t} + \frac{\partial}{\partial x} (\rho h u) = 0, \tag{2}$$

$$u = -\frac{k}{\mu} \frac{\partial P}{\partial x}. \tag{3}$$

In Eqs. (1)-(3) it is assumed that the filtering of the gas in a porous medium obeys Darcy's linear law [1, 2], that the process of heat exchange of the gas with the surface of the grains occurs instantaneously, and that the temperature of the effectively heated fraction of the porous medium per unit volume is constant. The correctness of the second condition follows from an evaluation of the characteristic time of establishing equilibrium between the gas temperature and the grain surfaces, which is obtained from the criterial dependence correlating the Nusselt, Prandtl, and Reynolds numbers [7, 8]. Generally speaking, the parameter m is a function of time. If the porous medium consists of sufficiently small grains, and the heating of the grains occurs within a time that is short compared with the characteristic times of the process under examination, then m is correlated with the porosity of the medium m_0 by the relation $m = 1 - m_0$. Otherwise, $m \leq 1 - m_0$.

Using the equation of state of gas

$$e = c_v T, \; h = c_p T, \; \rho = P(c_p - c_v)^{-1} T^{-1}, \tag{4}$$

we obtain from (1)-(3) a system of equations of filtration with instantaneous heat exchange and heating of the porous medium

$$\frac{\partial}{\partial t}\left(\frac{P}{T}\right) = \frac{k}{m_0 \mu} \frac{\partial}{\partial x}\left(\frac{P}{T} \frac{\partial P}{\partial x}\right), \tag{5}$$

$$\frac{\partial P}{\partial t} + \frac{m(c_p - c_v)\rho_s}{m_0 c_v} \frac{\partial T}{\partial t} = \frac{k c_p}{\mu c_v} \frac{\partial}{\partial x}\left(P \frac{\partial P}{\partial x}\right). \tag{6}$$

We write the initial and boundary conditions for (5) and (6) in the form

$$P(x, 0) = 0; \; P(0, t) = P_0; \; \lim_{x \to \infty} P(x, t) = 0; \; T(x, 0) = T_0. \tag{7}$$

The similarity variable of the problem (5)-(7) is the dimensionless magnitude $\theta = 0.5x(m_0\mu)^{1/2}(kP_0 t)^{-1/2}$.

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Taking the dimensionality into account, the gas pressure and temperature are represented in the form

$$P = P_0 f(\theta),\ T = T_0 \varphi(\theta). \tag{8}$$

If we substitute (8) into the system of equations (5), (6), we obtain a system of ordinary differential equations with respect to f and φ:

$$2\theta \frac{d}{d\theta}\left(\frac{f}{\varphi}\right) + \frac{d}{d\theta}\left(\frac{f}{\varphi}\frac{df}{d\theta}\right) = 0, \tag{9}$$

$$2\beta \frac{(c_v - c_p)}{c_v} \theta \frac{d\varphi}{d\theta} = \frac{c_p}{c_v}\frac{d}{d\theta}\left(f\frac{df}{d\theta}\right) + 2\theta \frac{df}{d\theta}. \tag{10}$$

In (10), β is a dimensionless magnitude, $\beta = c_S \rho_S m T_0 (m_0 P_0)^{-1}$.

Concerning the solution of the system of differential equations (9), (10), we make a precising assumption: we assume that there is such $\theta = \alpha$ for which

$$f(\alpha) = 0,\ \varphi(\alpha) = 1. \tag{11}$$

For $\theta = \alpha$, we obtain from (7):

$$f(0) = 1. \tag{12}$$

The system of equations (9), (10) may be represented in the form

$$2\theta\left(\frac{df}{d\theta} - f\frac{d\ln\varphi}{d\theta}\right) + f\frac{d^2 f}{d\theta^2} - f\frac{df}{d\theta}\frac{d\ln\varphi}{d\theta} + \left(\frac{df}{d\theta}\right)^2 = 0, \tag{13}$$

$$2\beta\theta(c_v - c_p)\frac{d\varphi}{d\theta} = c_p\left(\frac{df}{d\theta}\right)^2 + c_p f\frac{d^2 f}{d\theta^2} + 2\theta c_v \frac{df}{d\theta}. \tag{14}$$

Following [2, 9], we will seek the solution of (13), (14) with conditions (11), (12) in the form of the series

$$f = \sum_{k=0}^{\infty} p_k(\alpha - \theta)^k;\quad \varphi = \sum_{k=0}^{\infty} r_k(\alpha - \theta)^k. \tag{15}$$

It follows from (15) that

$$\left.\frac{df}{d\theta}\right|_{\theta=\alpha} = -p_1,\ \left.\frac{d^2 f}{d\theta^2}\right|_{\theta=\alpha} = 2p_2,\ \ldots,\ \left.\frac{d^k f}{d\theta^k}\right|_{\theta=\alpha} = k!(-1)^k p_k;$$

$$\left.\frac{d\varphi}{d\theta}\right|_{\theta=\alpha} = -r_1,\ \left.\frac{d^2\varphi}{d\theta^2}\right|_{\theta=\alpha} = 2r_2,\ \ldots,\ \left.\frac{d^k\varphi}{d\theta^k}\right|_{\theta=\alpha} = k!(-1)^k r_k. \tag{16}$$

Since $f(\alpha) = 0$ and $\varphi(\alpha) = 1$, $p_0 = 0$, $r_0 = 1$. Using (16), we determine from (13), (14) that $p_1 = 2\alpha$, $r_1 = 2\alpha/\beta$. If we differentiate (13), (14) with respect to θ, we find p_k and r_k:

$$p_2 = -\frac{1}{2},\ r_2 = -\frac{1}{2\beta};$$

$$p_3 = \frac{1}{36\alpha} + \frac{2\alpha}{9\beta},\ r_3 = \frac{1}{36\alpha\beta} + \frac{8\alpha(4c_p - c_v)}{36\beta^2(c_p - c_v)}.$$

For $T_0 = 400°K$; $c_S = 10^3$ J/(kg·°K); $\rho_S = 2.5 \cdot 10^3$ kg/m^3; $P_0 = 10^7$ N/m^2; $m/m_0 = 0.1$; $c_p/c_v = 4/3$, we find that $\beta = 10$.

The solution of the system of equations (9), (10) with conditions (11), (12), in the third approximation has the form

$$f = 2\alpha(\alpha - \theta) - \frac{1}{2}(\alpha - \theta)^2 + \left(\frac{1}{36\alpha} + \frac{\alpha}{45}\right)(\alpha - \theta)^3 + \ldots, \tag{17}$$

$$\varphi = 1 + \frac{\alpha}{5}(\alpha - \theta) - \frac{1}{20}(\alpha - \theta)^2 + \left(\frac{1}{360\alpha} + \frac{13\alpha}{45000}\right)(\alpha - \theta)^3 + \ldots \tag{18}$$

Taking into account that f(0) = 1, we obtain from (17):

$$1 = 2\alpha^2 - \frac{\alpha^2}{2} + \frac{\alpha^2}{36} + \frac{\alpha^4}{45}, \tag{19}$$

and hence α = 0.8521.

With α = 0.8571, the similarity solution of the problem of motion in a porous medium, without taking heat exchange into account ($\beta = \infty$), coincides with the solution given in [2].

It follows from a comparison of the solutions that heat exchange of gas does not substantially affect the pressure of the gas that is being filtered.

Using Darcy's linear law (3) and differentiating (17) with respect to x, we find the speed with which the gas moves in the porous medium:

$$u(x, t) = \frac{\sqrt{kP_0}}{2\sqrt{m_0\mu}}\left(\frac{\alpha}{\sqrt{t}} + \frac{x\sqrt{m_0\mu}}{2t\sqrt{kP_0}}\right). \tag{20}$$

In conclusion, we will discuss how the system of equations (1)-(3) with conditions (7) corresponds to the real motion of gas in a porous medium. A more rigorous solution of the problem (for all x and t) has to be carried out by using the gasdynamic system of equations taking frictional forces into account. However, a comparison of the numerical solutions of the filtering and the gasdynamic systems of equations showed that for the case examined within the framework of the present article, they practically do not differ from each other.

NOTATION

ρ, density; u, speed; μ, viscosity; e, energy; h, enthalpy; P, pressure of the filtered gas; m_0, porosity; k, permeability of the porous medium; m, fraction of the effectively heated part of the medium; e_s, energy of unit volume of the porous medium; T, temperature of the gas and of the medium; T_0, initial temperature of the medium; P_0, gas pressure at the boundary of the porous medium; θ, similarity variable; f, dimensionless gas pressure; φ, dimensionless gas temperature; c_p, c_v, heat capacities of the gas; α and β, parameters; p_k and r_k, coefficients of the series.

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